Labor allocation and development*

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Abstract

This paper proposes a new mechanism based in the allocation of labor to help to understand why differences among countries have remained stable. We pose a neoclassical growth model in which agents work but also devote time to commit predation. It is shown that when the elasticity of substitution between labor and capital is lower than one, the labor share rises along the transition when the per capita capital is lower than the steady state level. This increase in the labor share reduces the incentive to predate and rises the incentive to work in the production. Empirical evidence supports these results: low per capita income countries have larger portions of predation and present lower labor shares. In this setting, it is proved that the standards effects of an increase in the productivity are amplified by the collateral effects in predation and so, the reallocation of labor. Finally, we also prove that an institutional improvement reduces predation and increases the income per capita.

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JEL Classification: O15, O17, O29, O30.

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1. Introduction

Differences among countries have remained stable even has increased in latest decades. There has been many efforts in the current macroeconomics research addressed to explain this fact. To this respect, new features as the nature and composition of the economic activities have been explored. It is well-known that in economies resources are devoted to both productive activities (production of goods and services) and to unproductive activities. Unproductive activities entail a group of activities that share the common feature of being profitable, but wasteful: they use produce resources to produce profits (i.e., income) but not goods (for example, property crime, robbery, fraud, begging, etc.). The empirical evidence suggests that the size of the unproductive sector is larger in low income countries. For example, the share of unproductive sector on GDP is 20.7% for Latin America, while it is 6.89% for United States [1]. Other example in the literature is Bourguignon (1999) that finds that the share of property crime on GDP is 0.5% for United States, while it is 1.5% for Latin America.

Similarly, recent literature on the labor share pattern has found significant differences among countries with richest countries showing a bigger labor share. Figure 1 shows the long term labor share for a wide sample of countries ordered by their per capita income relative to the world average. Clearly, there is a positive correlation between the position of each country relative to the per capita income and the labor share reported.

This paper shows that countries which present a relative low labor share, are also countries with a large unproductive and predator sector. If the labor share is small enough then the agents' incentives to participate in the productive sector are

[1] Based in Anderson (1999) it is possible to calculate the total cost of the unproductive sector (non-violent diversion crimes) in United States as 6.89% of GDP. In the same way, from the estimations of Londoño and Guerrero (1998), the total cost of diversion in Latin America can be shaped to 20.7% of GDP.

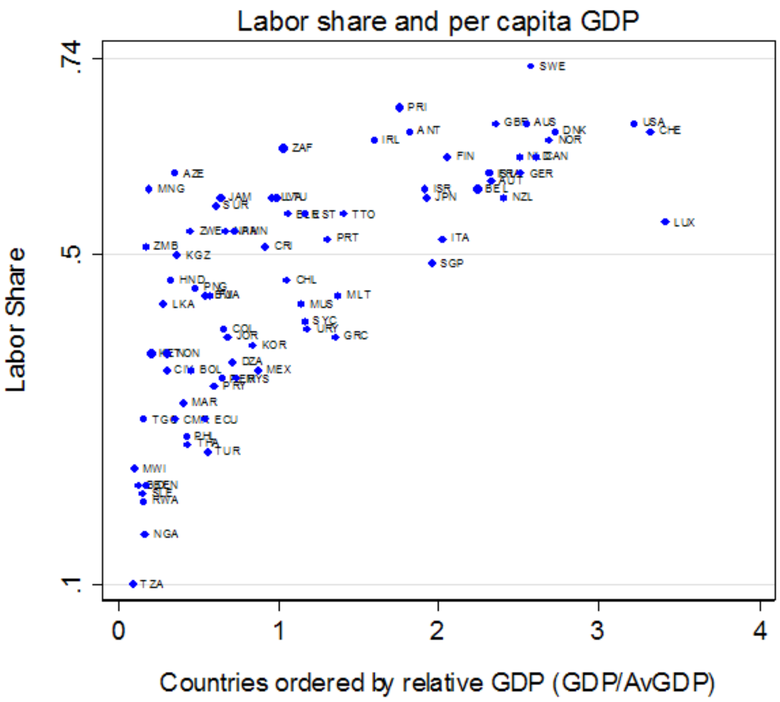

Figure 1: **Labor share and per capita GDP**

reduced. Agents devote their resources to participate in the unproductive sector. A reinforced predatory sector takes agents away from the productive sector and holds them trapped doing unproductive activities. The paper presents a model in which workers devote time to legal activities (output production and avoidance production) and illegal activities (predation). Given the fact that productive income is vulnerable to predation, agents have the possibility to protect themselves from it producing avoidance services. The equilibrium may shape in different types: for example, there may be a poor equilibrium where legal production pays little because predation is so common. There is also a good equilibrium with little predation, because legal production has a high payoff and deters almost all predation.

To understand the relationship between the labor share and the size of the unproductive sector is crucial to understand why countries are different. In fact, poor countries usually show relatively low labor shares and large unproductive sectors. It is easy to think that one possible reason explaining a low level of per capita income is the lack of incentives to undertake productive activities when predation in common.

Literature has been focused in the role of social institutions to deter predation. In this regard, a number of authors have developed theoretical models of equilibrium when protection against predation is incomplete (see for example, Murphy, Shleifer, and Vishny, 1991, Acemoglu, 1995, Schrag and Scotchmer 1993, Ljungqist and Sargent, 1995 and Grossman and Kim, 1996). In these models workers choose between production and diversion and social protective institutions are designed. They first define the bad equilibrium where diversion has a high payoff compared to the productive sector, because enforcement is ineffective when diversion is common. Then they define the good equilibrium with little diversion where production has a high payoff and the high probability of punishment deters almost all diversion..

The paper is organized as follows. Section 2 develops a model of two sectors: legal (production and avoidance) and illegal activities (predation). Section 3 derives the agents' decisions. Section 4 defines the equilibrium. Section 5 defines some technical concepts. Section 6 presents the dynamic behavior of the economy. Section 7 analyzes the role of the predation explaining the per capita GDP differences due to a shift in productivity. Section 8 studies the effect of change in the quality of the institutions which reduces the efficiency of the predation technology and, the last section concludes.

2. The model

Time is continuous with an infinite horizon. The economy is populated for many identical dynasties of homogeneous agents. There is a single good in the economy, which can be used for consumption and investment in physical capital:

$$Y(t) = C(t) + \dot{K}(t) + \delta K(t) \tag{2.1}$$

where $Y(t)$ denotes aggregate production, $C(t)$ denotes aggregate consumption, $K(t)$ aggregate capital and $\delta \in (0,1)$ denotes the depreciation rate. $\dot{K}(t) + \delta K(t)$ is the gross investment.

2.1. Technology

The production technology of this good is given by the following production function:

$$Y(t) = F(K(t), L(t)) \tag{2.2}$$

where K denotes physical capital and L denotes the labor. The production function, $F : \Re^2_+ \rightarrow \Re_+$, is assumed to be to be continuous, increasing (in both arguments), strictly cuasi-concave and presents constant return to scale. Furthermore, it is twice-continuously differentiable and strictly increasing in $\Re^2_{++}$ and; one zero input implies no output, $F(0,K) = F(K,0) = F(0,0) = 0$ and Inada conditions are satisfy $\lim_{\kappa \rightarrow 0} F'_K(\kappa, 1) = +\infty$ and $\lim_{\kappa \rightarrow +\infty} F'_K(\kappa, 1) = 0$ for $\kappa \equiv K/L$. Finally, the production function $F(.)$ is assumed to exhibit an elasticity of substitution between its inputs less than one:

$$\forall \kappa \in \Re_{++} \quad \sigma^f(\kappa) \equiv \frac{\partial \ln(\kappa)}{\partial \ln\left(RMST_{L,K}(\kappa, 1)\right)} = \frac{1}{\frac{\partial RMST_{L,K}(\kappa,1)}{\partial \kappa} \frac{\kappa}{RMST_{L,K}(\kappa,1)}} < 1$$

where $\sigma^f(\kappa)$ is the elasticity of substitution between labor and capital (of the production function), which depend on the capital-labor ratio $\kappa \equiv K/L$. The assumption that the elasticity of substitution is lower than one implies that the labor share increases with the capital-labor ratio. This property will play a key role in our results.

2.2. Preferences

The preferences of a dynasty are given by a time separable, constant elasticity of substitution utility function:

$$\int_0^\infty u(c(t))e^{-\rho t}dt \tag{2.3}$$

where $c(t)$ denotes the per capita consumption of dynasty in period t and $\rho > 0$ is the discount rate of the utility function. Agents do not differentiate between the consumption of different members of the dynasty, they only care about their aggregate consumption.

2.3. The predation technology

Each period agents are endowed with 1 unit of time which can be devoted to undertake two types of economic activities: to produce final goods (legal sector), l and to undertake predation (illegal sector) l_s, that is,

$$1 = l(t) + l_s(t) \tag{2.4}$$

For predation activities we understood all the activities which imply the appropriation of uncompensated value from others improperly, that is, without permission, without making any contribution to productivity or with intentional deceptions. We are thinking in property crimes, fraud, corruption, etc. The non-market income of an agent who is engaged in predation is denoted by $\widetilde{y}(t)g\left(l_s\right)$, where $\widetilde{y}(t)$ is the per capita gross income of agents who work in the production sector and $g : \Re_+ \to [0, 1]$ is the fraction of per capita gross income that each agent obtains when devotes time to predation, which depend positively in the amount of time devoted to such activity l_s. We assume that the function $g(.)$ is strictly increasing, strictly concave, continuous and differentiable of second order and $g(0) = 0$, $g(1) < 1$ and $g'(0) \geq 1$.

3. Agents' decisions

3.1. Households:

The household' maximization problem is as follows:

$$\max_{\{c(t),l(t),l_s(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} u(c(t))e^{-\rho t}dt \tag{3.1}$$

$$\begin{aligned} s.a \quad : \quad \dot{b}\ (t) &= \underbrace{w(t)l(t) + r(t)b(t) - g(\widetilde{l}_s(t))y(t)}_{\text{Net Income from the production sector}} + \underbrace{g(l_s(t))\widetilde{y}(t)}_{\text{Income from predation}} - c(t) \\ l(t) + l_s(t) &= 1 \\ y(t) &= w(t)l(t) + (\delta + r(t))b(t) \end{aligned}$$

where $b(t)$ denotes the assets of the household, $w(t)$ is the wage per unit of labor, $r(t)$ the *net* return to assets and $y(t)$ is the household's *gross* income. The sign " ~" over a variable means that this variable is a per capita variable of the economy and therefore the household cannot decide over it. Thus, $\widetilde{l}_s$ denotes per capita labor devoted to predation and $\widetilde{y}$ per capita gross income. Income from the production sector is equal to labor income from the production sector $w(t)l(t)$ plus financial income $r(t)b(t)$ minus the amount of this income that is predated by other agents in the economy $g(\widetilde{l}_s(t))y(t)$. The other source of income is coming from the predation sector which is equal to $g(l_s(t))\widetilde{y}(t)$. The increase of the household's assets $\dot{b}\ (t)$ is equal to her saving, which is equal to her income (the one from production plus the one from predation) minus consumption $c(t)$.

The first order conditions for interior solution imply the following conditions:

$$w(t)\left[1 - g(\widetilde{l}_s(t))\right] = g'_{l_s}(l_s(t))\widetilde{y}(t) \tag{3.2}$$

$$\frac{\dot{c}\ (t)}{c(t)} = \frac{1}{\sigma^u(c(t))}\left[(r(t) + \delta)\left(1 - g(\widetilde{l}_s(t))\right) - \delta - \rho\right] \tag{3.3}$$

where $\sigma^u(c(t)) = -\frac{u''(c)c}{u'(c)}$ is the elasticity of the marginal utility. The first of the above condition (equation 3.2) says that the net wage in the production sector after predation should be equal to the marginal payment of predation activities. That is, the marginal payment of the time devoted to each activity should be equal. Equation (3.3) is the typical Euler equation. The speed at which consumption grows depends positively on return on savings, $(r(t)+\delta)\left(1-g(\widetilde{l}_s(t))\right)-\delta$ and negatively in the patient rate of the household, ρ. Finally, the more concave the utility function (the higher $\sigma^u(c(t))$), the more smooth the consumption path.

The following transversality condition should be also satisfied:

$$\lim_{t\to+\infty} u'(c(t))e^{-\rho t}b(t)=0$$

3.2. Firms:

Firms maximize profits:

$$\max_{k,l^d} F(k,l^d)-wl^d-(\delta+r)k \tag{3.4}$$

where k denotes the per capita capital.

The First order conditions of the above problem are:

$$\begin{aligned} F'_k(k,l^d) &= f'(\kappa)=(\delta+r) \\ F'_L(k,l^d) &= f(\kappa)-f'(\kappa)\,\kappa=w \end{aligned}$$

where $\kappa=k/l^d$. The above conditions are the well known and says that the firm hire a factor until reaching the point in which the marginal productivity of this factor is equal to its price.

4. Equilibrium Definition

The equilibrium definition is standard: equilibrium occurs when agents maximize their objective functions and markets clear. Since all households are alike, we may define equilibrium in per capita terms.

Definition 4.1. *An equilibrium is an allocation* $\left\{c(t), l(t), l_s(t), b(t), l^d(t), k(t), \widetilde{l}_s(t), \widetilde{y}(t)\right\}_{t=0}^{\infty}$ *and a vector of prices* $\{w(t), r(t)\}_{t=0}^{\infty}$.

- Households maximize her utility, that is, $\{c(t), l(t), l_s(t), b(t)\}_{t=0}^{\infty}$ is the solution of the household's maximization problem (3.1)
- Firms maximize profits, that is, $\forall t\ l^d(t), k(t)$ is the solution of the optimization problem of firms (3.4).
- Capital market clears: $\forall t\ k(t) = b(t)$
- Labor market clears: $\forall t\ l^d(t) = l(t)$
- Finally, since households are identical, per capita variables coincide with household variables: $\forall t\ \widetilde{l}_s(t) = l_s(t)$ and $\widetilde{y}(t) = w(t)l(t) + (\delta + r(t))b(t)$.

Definition 4.2. *Steady state equilibrium is an equilibrium in which both the allocation and the prices remain always constant over time.*

5. Predation and per capita capital

5.1. Labor share and capital-labor ratio

Let's denote $f(\kappa) = F(\kappa, 1)$ the production per efficient unit of labor, which depends on the capital-labor ratio $\kappa \equiv K/L$, and the capital share, $\alpha(\kappa) \equiv \frac{f'(\kappa)\kappa}{f(\kappa)}$. Remember that it was assumed that the elasticity of substitution between labor and capital were less that one:

$$\sigma^f(\kappa) = \frac{1}{\frac{\partial RMST_{L,K}(\kappa,1)}{\partial \kappa}\frac{\kappa}{RMST_{L,K}(\kappa,1)}} = \frac{1}{\frac{\partial\left(\frac{f(\kappa)-f'(\kappa)\kappa}{f'(\kappa)}\right)}{\partial \kappa}\frac{\kappa}{\frac{f(\kappa)-f'(\kappa)\kappa}{f'(\kappa)}}} = \frac{1-\alpha(\kappa)}{\frac{-f''(\kappa)\kappa}{f'(\kappa)}} < 1 \tag{5.1}$$

It is easy to prove that the assumption that the elasticity of substitution between labor and capital is lower than one implies that the labor share increases with the capital-labor ratio:

$$\begin{aligned}\frac{\partial\left(1-\alpha(\kappa)\right)}{\partial\kappa} &= \frac{\partial\left(\frac{f(\kappa)-f'(\kappa)\kappa}{f(\kappa)}\right)}{\partial\kappa}=\frac{-f''(\kappa)\kappa}{f(\kappa)}-\frac{\left[f(\kappa)-f'(\kappa)\kappa\right]f'(\kappa)}{\left[f(\kappa)\right]^{2}}= \\ &= \frac{f'(\kappa)}{f(\kappa)}\left(1-\alpha(\kappa)\right)\left[\frac{1-\sigma^{f}(\kappa)}{\sigma^{f}(\kappa)}\right]\;>0\end{aligned}$$

5.2. Labor devoted to predation and labor share

Using equation (3.2) and the fact that all household are identical ($\widetilde{l}_s = l_s$), it follows that:

$$\phi(l_s)=\frac{g'(l_s)(1-l_s)}{\left[1-g(l_s)\right]}=1-\alpha \tag{5.2}$$

where $\phi:[0,1]\rightarrow\Re_{+}$ is defined as $\phi(x)=\frac{g'(x)(1-x)}{1-g(x)}$.

Lemma 5.1. *$\phi(1)=0$ and there is $l_s^{\min}\in[0,1)$ such that $\phi\left(l_s^{\min}\right)=1$ and such that $\phi(.)$ is strictly decreasing in $\left[l_s^{\min},1\right]$ and $\phi(l_s)>1$ when $l_s<l_s^{\min}$.*

Figure 2.a displays function $\phi(l_s)$. It follows from the Implicit Function Theorem that l_s is an decreasing function of the labor share:

$$\frac{\partial l_s}{\partial(1-\alpha)}=\frac{1}{\underbrace{\phi'(l_s)}_{\ominus}}<0$$

Obviously, the amount of labor devoted to production is an increasing function of the labor share:

$$\frac{\partial l}{\partial(1-\alpha)}=-\frac{\partial l_s}{\partial(1-\alpha)}>0$$

We may summarize this result in the following corollary:

Corollary 5.2. *The portion of labor devoted to predation l_s is a strictly decreasing function of the labor share, being $l_s = 1$ when $1 - \alpha = 0$ and $l_s = l_s^{\min} \leq 1$ when $1 - \alpha = 1$.*

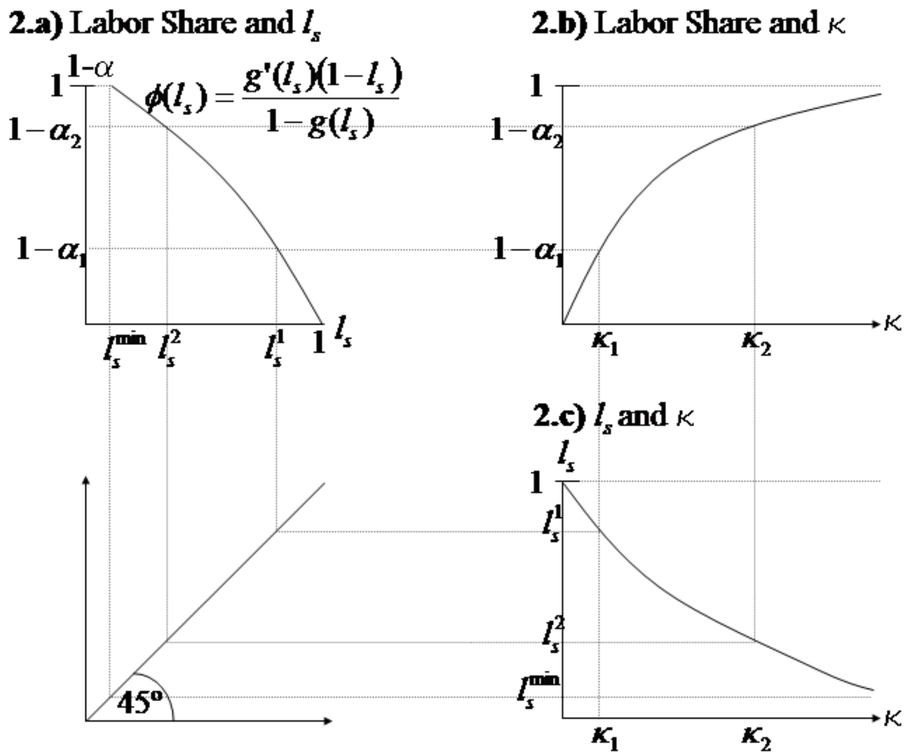

Figure 2: **Labor devoted to predation and capital-labor ratio**

5.3. Labor devoted to predation and capital-labor ratio

Since the amount of labor devoted to predation decreases with the labor share and the labor share increases with the capital-labor ratio, we conclude that the amount of labor devoted to predation decreases with the capital-labor ratio.

Proposition 5.3. *The portion of labor devoted to predation at equilibrium* l_s *is a strictly decreasing function of the capital-labor ratio in the production sector. The portion of labor devoted to production at equilibrium* l *is a strictly increasing function of the capital-labor ratio. At equilibrium* $l_s \in \left(l_s^{\min}, 1\right)$ *and* $l \in (0, l^{\max})$ *where* $l^{\max} \equiv 1 - l_s^{\min} \in (0, 1)$.

Figure 2.b shows that when the capital-labor ratio rises in the production sector (from κ_1 to κ_2), due to the elasticity of substitution lower than one, the labor share rises as well (from $1-\alpha_1$ to $1-\alpha_2$). This reduces the household's incentives to devote time to predation as figure 2.a shows (passing the labor devoted to predation from l_s^1 to l_s^2). Figure 2.c shows that the rise of the capital-labor ratio rises in the production sector (from κ_1 to κ_2) generates a drop in the labor devoted to predation (passing from l_s^1 to l_s^2).

From now on we will call $l_s(\kappa)$ the function that relates the amount of labor devoted to predation in equilibrium with the capital-labor ratio in the production sector, and $l(\kappa)$ the function that relates the amount of labor devoted to production in equilibrium with the capital-labor ratio in the production sector.

6. Dynamic Behavior

6.1. Dynamic system

It follows from the equilibrium definition that the dynamic behavior of capital is given by the following equation:

$$\dot{k}\,(t) = F\left(k(t), l(\kappa(t))\right) - c(t) - \delta k(t)$$

The above equation yields the following accumulation equation of the capital-labor ratio in the production sector:

$$
\begin{aligned}
\dot{\kappa}\ (t) &= \frac{\dot{k}\ (t)}{l(t)} - \kappa(t)\frac{\dot{l}\ (t)}{l(t)} \\
\dot{\kappa}\ (t) &= f\left((\kappa(t))\right) - \frac{c(t)}{l(\kappa(t))} - \delta\kappa(t) - \frac{l'(\kappa(t))\kappa(t)}{l(\kappa(t))}\ \dot{\kappa}\ (t) \\
\dot{\kappa}\ (t) &= \frac{f\left((\kappa(t))\right) - \frac{c(t)}{l(\kappa(t))} - \delta\kappa(t)}{1 + \frac{l'(\kappa(t))\kappa(t)}{l(\kappa(t))}}
\end{aligned}
$$

It follows from the Euler equation (3.3) that:

$$
\frac{\dot{c}\ (t)}{c(t)} = \frac{1}{\sigma^u(c(t))}\left[f'(\kappa(t))\left[1 - g\left(l_s(\kappa(t))\right)\right] - \delta - \rho\right]
$$

Thus, the dynamic behavior of the economy may be characterized with the following dynamic system:

$$
\dot{\kappa}\ (t) = \frac{f\left((\kappa(t))\right) - \frac{c(t)}{l(\kappa(t))} - \delta\kappa(t)}{1 + \kappa(t)\frac{l'(\kappa(t))}{l(\kappa(t))}} \tag{6.1}
$$

$$
\frac{\dot{c}\ (t)}{c(t)} = \frac{1}{\sigma^u(c(t))}\left[f'(\kappa(t))\left[1 - g\left(l_s(\kappa(t))\right)\right] - \delta - \rho\right] \tag{6.2}
$$

$$
\lim_{t\to+\infty} u'(c(t))e^{-\rho t}\kappa(t)l(\kappa(t)) = 0 \tag{6.3}
$$

It follows from (6.2) that in order to analyze the dynamic behavior of the economy it is important to understand the way in which the return on savings evolves with the capital-labor ratio. The following proposition establishes that the return on savings is a decreasing function as in the neoclassical model.

Proposition 6.1. *The net return on savings (after predation),* $f'(\kappa)\left[1 - g\left(l_s(\kappa(t))\right)\right] - \delta$*, is a decreasing function of* κ *and* $\lim_{\kappa\to 0} f'(\kappa)\left[1 - g\left(l_s(\kappa(t))\right)\right] - \delta = +\infty$ *and* $\lim_{\kappa\to+\infty} f'(\kappa)\left[1 - g\left(l_s(\kappa(t))\right)\right] - \delta = -\delta$.

Note that when the capital-labor ratio in the production sector rise, the marginal rate of the capital $f'(\kappa)$ goes down but the portion of income that goes to factors after predation $[1 - g(l_s(\kappa(t)))]$ goes up. Thus, there are two opposite mechanisms determining the evolution of the return to savings. However, the above proposition establishes that the return to savings always decreases with capital in spite of the increasing portion of income that goes to factors after predation.

Corollary 6.2. *There is a unique steady state with positive amount of capital.*

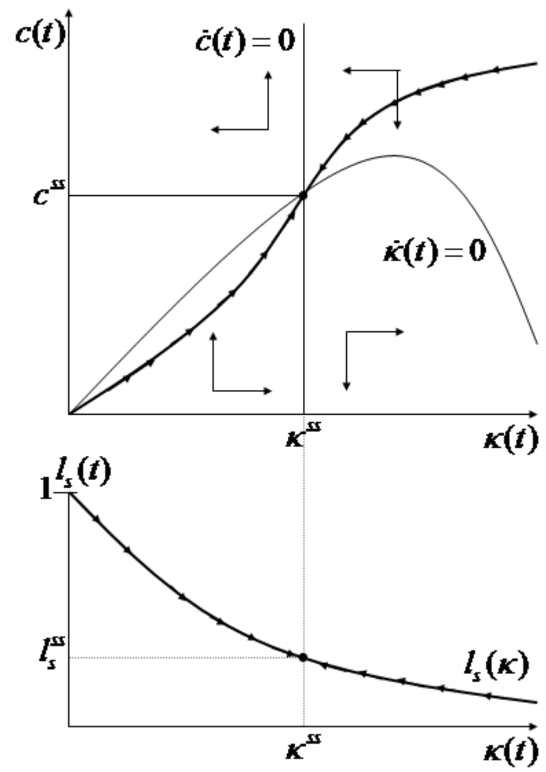

Figure 3: **Dynamic Behavior**

Phase diagram in figure 3 shows that the dynamic behavior of the economy is characterized by the typical saddle point dynamic: there is a unique path which converges to the steady state. This means, that given the initial level of per capita capital[2], there is a unique equilibrium path, which converges to the steady state. When the initial amount of per capita capital is lower than the steady state level, the capital-labor ratio, the consumption and the portion of labor devoted to production grows along the equilibrium path, converging to their steady state levels, while the labor devoted to predation goes down. When the amount of per capita capital is larger than the steady state level the opposite happens.

7. Predation as an amplification mechanism of differences in productivity

Many authors have emphasized the key role of differences in productivity to understand differences in per capita income across countries (see See Easterly and Levine, 2001, Hall and Jones, 1999, and Parente and Prescott, 2000). In this section we are going to modify the model to introduce differences in productivity across countries. More precisely, we are going to consider than production depends on a parameter A which is an index of total factor productivity:

$$Y(t) = AF\left(K(t), L(t)\right)$$

where $F\left(K, L\right)$ satisfies all the assumptions presented in Section 2. Note that this modification of the model do not affect the relationship between the capital-labor ratio in production and the labor share. Thus, do not affect the relationship between capital-labor ratio in production and labor devoted to predation. Thus,

[2]Note that the capital-labor ratio in the production sector is an increasing function of per capita capital and vice versa:
$\kappa = \frac{k}{l(\kappa)} \Leftrightarrow \kappa l(\kappa) - k = 0 \Rightarrow \frac{\partial \kappa}{\partial k} = \frac{1}{l(\kappa)+\kappa l'(\kappa)} > 0$

the dynamic system that describes the behavior of the economy (see equations 6.1 - 6.3) is as follows:

$$\dot{\kappa}\,(t) = \frac{Af\left((\kappa(t))\right) - \frac{c(t)}{l(\kappa(t))} - \delta\kappa(t)}{1 + \kappa(t)\frac{l'(\kappa(t))}{l(\kappa(t))}} \tag{7.1}$$

$$\frac{\dot{c}\,(t)}{c(t)} = \frac{1}{\sigma^u(c(t))}\left[Af'(\kappa(t))\left[1 - g\left(l_s(\kappa(t))\right)\right] - \delta - \rho\right] \tag{7.2}$$

$$\lim_{t\to+\infty} u'(c(t))e^{-\rho t}\kappa(t)l(\kappa(t)) = 0 \tag{7.3}$$

The effect of an increase in the total factor productivity index A is displayed in Figure 4. Such increase makes the locus $\dot{\kappa}\,(t) = 0$ to go up and the locus $\dot{c}\,(t) = 0$ to move right. Thus, the capital-labor ratio and the amount of labor devoted to production at the steady state go up. This involve an increase in per capita income $y^{ss} = Af(\kappa^{ss})l(\kappa^{ss})$ at the steady state. The facts that there is predation in the model and that the labor devoted to predation falls with the capital-labor ratio, amplify the effect of the rise of productivity on per capita income at the steady state due to three mechanisms:

1. The rise in productivity increases the return to savings, rising the incentive to accumulate capital (see Euler equation 7.2). When capital grows labor devoted to predation falls, reducing the portion of income that goes to predation $g\left(l_s(\kappa(t))\right)$, and this increases the return on savings producing an amplification effect on the increase of capital due to the rise in productivity.

2. Furthermore, the return to savings not only increases due to the fall in the part of income that goes to predation. There exists an additional amplification effect due to the fact that when capital increases due to the rise in productivity, the portion of labor devoted to production increases, and this implies an increase in the marginal return to capital and the return to saving amplifying further the effect of productivity on per capita capital.

3. Finally, the amplification effect over the per capita income is not only due to the amplification effect over the per capita capital. There is an additional direct effect of the increase in the amount of labor devoted to production on the

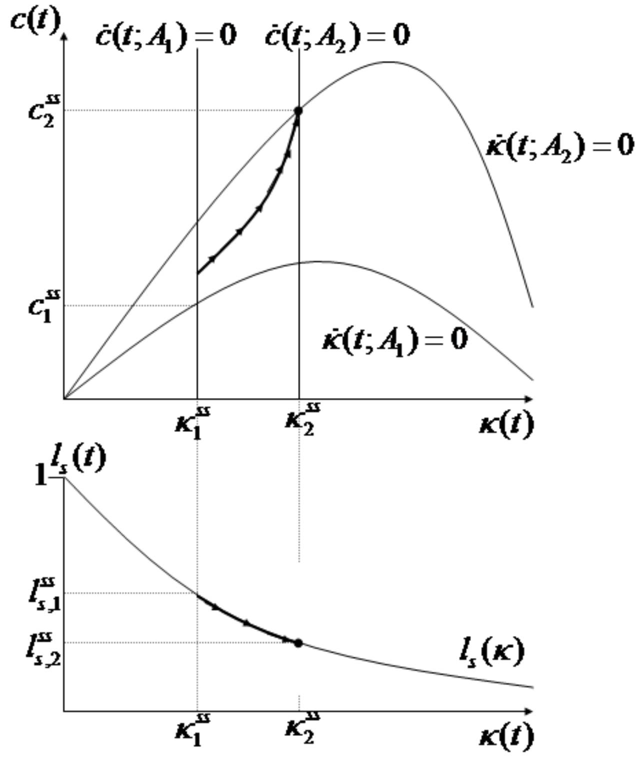

Figure 4: **Effect of an increase in the productivity**

Figure 5 shows the tree mechanism described above. Figure 5.a splits the effect of the rise on productivity on the capital-labor ratio κ in two parts: i) the standard effect, indicated with number *1*, which is the effect of an increase of productivity over the capital-labor ratio, keeping the portion of labor devoted to production, and therefore, the portion of income that is predated constant (standard effect). ii) The amplification effect over the capital- labor ration κ due to predation, indicated with number *2*, which is equal to the increase of the capital-labor rate due to the reduction in the portion of the payment of capital that goes to predation. Thus, number *2* in figure 5.a displays the effect of mechanism *1* exposed above: the reduction in the labor devoted to predation, reduces the portion of the payment of capital that goes to predation, increasing the return to savings and the incentive to save, promoting the accumulation of capital.

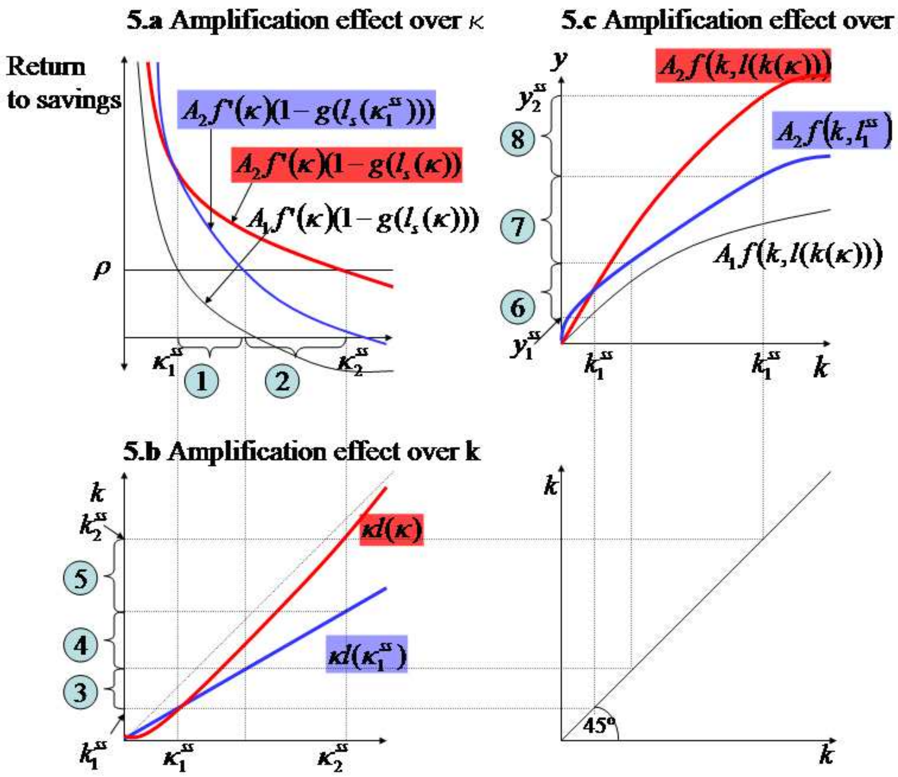

Figure 5: **Amplification effect of an increase in the productivity**

Figure 5.b split the effect of an increase in the productivity over per capita capital in three: i) number *3* display the standard effect that occur when the portion of labor devoted to production and predation remain constant (at the

initial level $l(\kappa_1^{ss})$ and $l_s(\kappa_1^{ss})$). ii) Number *4* displays the amplification effect due to the reduction in the portion of the payment to capital that goes to predation, that came from 5.a. This effect is referred to mechanism *1* exposed above. iii) Number *5* display mechanism *2* explained above. When the capital increases in response to an increase on productivity, the labor share rises as well, increasing the agents' incentives to devote a larger portion of their labor to production, rising the marginal productivity of capital and the return to savings, and fostering the accumulation of capital.

Figure 5.c split the effect of an increase in productivity over per capita income in three: i) standard effect (number *6*). ii) The amplification effects of predation on per capita income due to the increase in the incentives to accumulate more capital, which are indexed by number *7* and are referred to mechanisms *1* and *2* explained above. iii) Finally, number *8* displays the effect on per capita income of mechanism *3*: the increase of productivity implies more capital accumulation which rises the labor share, encouraging agent to devote more time to production, which have a positive direct effect over production.

It is shown in the appendix 10.4 that the effect of the increase in the productivity over per capita income may split in two parts: the standard effect and the amplification effect due predation (due to the three mechanism described above), this is,

$$\frac{\partial y^{ss}}{\partial A}\frac{A}{y^{ss}} = \underbrace{1+\frac{\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}{1-\alpha(\kappa^{ss})}}_{\text{Standard Effect}} + \underbrace{\frac{\alpha(\kappa^{ss})\left(1\text{-}\sigma^f(\kappa^{ss})\right)}{1\text{-}\alpha(\kappa^{ss})}\left[\frac{1+\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}{-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))}+\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}\right]}_{\text{Amplification Effect due to predation}}$$

Note that the amplification effect only occurs when the elasticity of substitution is lower than one. This assumption plays a key role in the amplification effect, since the increase in capital only raises the labor share and the incentives to devote more labor to production when the elasticity of substitution is lower than one.

8. Institutional quality

Many authors have shown the empirical relevance of differences in institutions to explain differences in per capita income (see Acemoglu, Johnson and Robinson (2005) for a complete survey). In this section we capture this empirical fact by modifying the production function of the predation sector that now is going to depend negatively of an index of institutional quality denoted by $\Gamma \in \Re_{+}$. Thus, an increase in the index of institutional quality reduces the productivity of the predation sector, discouraging the use of labor in that sector and encouraging the use of labor in production. To be more precise, we assume that the fraction of per capita gross income that each agent obtains when devotes time to predation is a function $g : \Re_{+}^{2} \rightarrow [0,1]$ which is continuous and differentiable of second order, strictly increasing strictly concave in its first argument, that is, $g'_{l_s}(l_s,\Gamma) > 0$, $g"_{l_s^2}(l_s,\Gamma) < 0$, $g(0,\Gamma) = 0$, $g(1,\Gamma) < 1$ and $g'(0,\Gamma) \geq 1$. Furthermore, we assume that $\forall l_s > 0$ $g'_{\Gamma}(l_s,\Gamma) < 0$ and $g"_{l_s,\Gamma}(l_s,\Gamma) < 0$.

Proposition 8.1. *The portion of labor devoted to predation at equilibrium l_s is a strictly decreasing function the index of institutional quality Γ.*

An increase in the index of institutional quality reduces the productivity of the predation technology and, therefore, the incentives to devote time to such activity.

The dynamic behavior of the economy may be characterized by the following dynamic system (see dynamic system 6.1-6.3):

$$\dot{\kappa}\ (t) = \frac{f\left(\left(\kappa(t)\right)\right)\left[1 - g\left(l_s(\kappa(t),\Gamma),\Gamma\right)\right] - \frac{c(t)}{l(\kappa(t),\Gamma)} - \delta\kappa(t)}{1 + \kappa(t)\frac{l'(\kappa(t),\Gamma)}{l(\kappa(t),\Gamma)}} \tag{8.1}$$

$$\frac{\dot{c}\ (t)}{c(t)} = \frac{1}{\sigma^u(c(t))}\left[f'(\kappa(t))\left[1 - g\left(l_s(\kappa(t),\Gamma),\Gamma\right)\right] - \delta - \rho\right] \tag{8.2}$$

$$\lim_{t\to+\infty} u'(c(t))e^{-\rho t}\kappa(t)l(\kappa(t),\Gamma) = 0 \tag{8.3}$$

Figure 6 shows the effect of an increase in the index of institutional quality Γ. Such increases makes the locus $\dot{\kappa}\ (t) = 0$ to go up and the locus $\dot{c}\ (t) = 0$ to move right. Thus, the capital-labor ratio and the amount of labor devoted to production go up at the steady state. Furthermore, the amount of labor devoted to predation relative to the capital-labor ratio goes down. Thus, a improvement in institutions reduces incentives to predation, increases the portion of labor devoted to production and the portion of the marginal productivity of capital that goes to savers, and thus, it fosters the capital accumulation.

The effect of the increase in Γ over per capita income is as follows (see appendix 10.6):

$$\frac{\partial y^{ss}}{\partial\Gamma}\frac{\Gamma}{y^{ss}} =$$
$$\alpha(\kappa^{ss})\frac{\sigma^f(\kappa^{ss})\left[-\varepsilon_\Gamma^{1-g}(\kappa^{ss},\Gamma)+(1-\alpha(\kappa^{ss}))\varepsilon_\Gamma^l(\kappa^{ss},\Gamma)\right]}{(1-\alpha(\kappa^{ss}))} + \varepsilon_\Gamma^l(\kappa^{ss},\Gamma)+$$
$$\frac{\left[1-\sigma^f(\kappa^{ss})\right]\sigma^f(\kappa^{ss})\left[-\varepsilon_\Gamma^{1-g}(\kappa^{ss},\Gamma)+(1-\alpha(\kappa^{ss}))\varepsilon_\Gamma^l(\kappa^{ss},\Gamma)\right]\alpha(\kappa^{ss})}{(1-\alpha(\kappa^{ss}))\sigma^f(\kappa^{ss})}\left[\frac{\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}{-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}\right.$$
$$\left.\left(\frac{1}{-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})}\right)\left(1+\frac{\alpha(\kappa^{ss})\left[1-\sigma^f(\kappa^{ss})\right]}{-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}\right)\right]$$

where $\varepsilon_\Gamma^{1-g}(\kappa,\Gamma) = -\frac{g'_\Gamma(l_s(\kappa,\Gamma),\Gamma)\Gamma}{1-g(l_s(\kappa,\Gamma),\Gamma)} > 0$ is the elasticity of the fraction of income that goes to production factors with respect to Γ, $\varepsilon_\Gamma^l(\kappa,\Gamma) = \frac{\partial l(\kappa,\Gamma)}{\partial\Gamma}\frac{\Gamma}{l(\kappa,\Gamma)}$ is the elasticity of labor with respect to Γ. Note that the last two lines of the above expression are multiply by $(1 - \sigma^f(\kappa^{ss}))$. Thus, these two lines represent an amplification effect of the improvement in Γ over per capita income due to the fact that the increase of the capital-labor ratio reduces the incentives to predation due to the increase in the labor share, which occurs when the elasticity of substitution is lower than one.

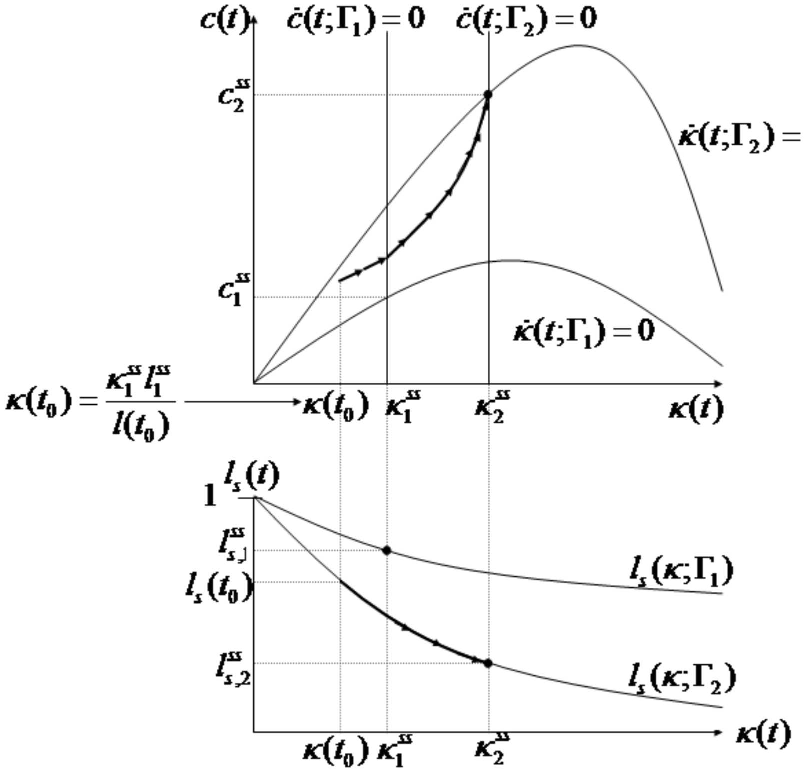

Figure 6: **Effect of an improvement in institutions**

9. Conclusion

This paper has presented a neoclassical growth model with predation in which the elasticity of substitution between labor and capital is lower than one. This property of the production function implies that labor share rises along the transition when the initial per capita capital is lower than the steady state level. This increase in the labor share implies a reduction in the incentive to predation and a reallocation of labor from predation to production along the transition.

We also analyze the amplification effect that predation may have on differences in productivity across countries. Even thought many authors have point out the differences in productivity as a source of differences in per capita income, these differences in productivity are not empirically high enough to generate the differences in per capita income across countries observed empirically. This paper propose a mechanism that amplifies the differences in per capita income generated by differences in productivity. When productivity rises, there is a direct effect on production and an indirect effect due to the accumulation of capital: the rise in productivity increases the return to savings and so, the incentives to accumulate more capital. In our model, together with these standard mechanisms, it appears other mechanism that amplifies the effect of productivity on per capita income which is related with predation and the assumption of the elasticity of substitution smaller than one. When productivity rises, the per capita capital rises, and this, due to the assumption of the elasticity of substitution lower than one, implies that the labor share increases, reducing the incentive to predation and increasing the portion of labor devoted to production. This increment in the amount of labor devoted to production has three effect: i) a direct effect on per capita production; ii) an indirect effect due to the accumulation of capital: when labor rises, it increases the marginal productivity of capital and the incentive to accumulate more capital; iii) finally, the reduction in the portion of labor devoted to predation implies that the share of the marginal product of capital that goes to savers increases, rising the return to savings and promoting the accumulation of capital.

Finally, we analyze the effect of an institutional change that reduces the productivity of the predation technology. Such change discourages predation increasing the portion of labor devoted to production. This increase in the labor devoted to production not only have a direct effect on production, it also encourages the accumulation of capital due to two mechanisms: i) it increases the marginal product of capital and therefore the return to savings; ii) it reduces the portion of the payments to capital that goes to predation, increasing the return to savings. Furthermore, when the capital-labor ratio rises, the labor share in the production sector increase, due to the assumption of elasticity of substitution lower than one, and this promote further the reallocation of labor from predation to production.

10. Appendix

10.1. Proof of Lemma 5.1

It was assumed that $g(1) < 1$ which implies $\phi(1) = \frac{g'(1)(1-1)}{[1-g(1)]} = 0$.

Note that if $l_s < 1$ and $\phi(l_s) \leq 1$ then

$$\phi'(l_s) = \frac{g"(l_s)(1-l_s)-g'(l_s)+\phi(l_s)g'(l_s)}{[1-g(l_s)]} \leq \frac{g"(l_s)(1-l_s)-g'(l_s)+g'(l_s)}{[1-g(l_s)]} = \frac{g"(l_s)(1-l_s)}{[1-g(l_s)]} < 0 \tag{10.1}$$

By assumption $g'(0) \geq 1$ and $g(0) = 0$, if $g'(0) = 1$, $\phi(0) = \frac{g'(0)(1-0)}{[1-g(0)]} = g'(0) = 1$ and in this case $l_s^{\min} = 0$; if $g'(0) > 1$, $\phi(0) = g'(0) > 1$, since $\phi(1) = 0$ it follows from continuity of $\phi(.)$ and from (10.1) that there is a unique $l_s^{\min}$ such that $\phi(l_s^{\min}) = 1$. It follows from (10.1) that when $l_s > l_s^{\min}$ then $\phi(l_s) < 1$ and $\phi'(l_s) < 0$. Finally, note that when $g'(0) > 1$ it follows from definition of $l_s^{\min}$ that $\forall l_s < l_s^{\min}$ $g(l_s) > 1$. ■

10.2. Proof of Proposition 5.3

$$\frac{\partial l_s}{\partial \kappa} = \underbrace{\frac{\partial l_s}{\partial (1-\alpha)}}_{\ominus} \underbrace{\frac{\partial (1-\alpha)}{\partial \kappa}}_{\oplus} = \frac{1}{\phi'(l_s)} \frac{f'(\kappa)}{f(\kappa)} (1-\alpha(\kappa)) \left[\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)} \right] < 0 \tag{10.2}$$

■

10.3. Proof of Proposition 6.1

$$\frac{\partial \left[f'(\kappa)[1-g(l_s(\kappa))] - \delta \right]}{\partial \kappa} = f'(\kappa)[1-g(l_s(\kappa))] \left[\frac{f"(\kappa)}{f'(\kappa)} - \frac{g'(l_s(\kappa))}{[1-g(l_s(\kappa))]} l_s'(\kappa) \right] =$$

substituting eqs. (5.2) and (10.2)

$$= f'(\kappa)[1-g(l_s(\kappa))] \left[\frac{f"(\kappa)}{f'(\kappa)} - \frac{\phi(l_s(\kappa))}{1-l_s(\kappa)} \frac{1}{\phi'(l_s(\kappa))} \frac{f'(\kappa)}{f(\kappa)} (1-\alpha(\kappa)) \left[\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)} \right] \right] =$$

substituting eq. (5.1)

$$= f'(\kappa)[1 - g(l_s(\kappa))] \left[-\frac{1-\alpha(\kappa)}{\sigma^f(\kappa)\kappa} - \frac{\phi(l_s(\kappa))}{1-l_s(\kappa)} \frac{1}{\phi'(l_s(\kappa))} \frac{f'(\kappa)}{f(\kappa)} (1-\alpha(\kappa)) \left[\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)} \right] \right] =$$

$$= \frac{f'(\kappa)[1 - g(l_s(\kappa))] (1-\alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[-1 - \frac{\phi(l_s(\kappa))}{1-l_s(\kappa)} \frac{1}{\phi'(l_s(\kappa))} \frac{f'(\kappa)\kappa}{f(\kappa)} \left[1 - \sigma^f(\kappa) \right] \right] =$$

$$= \frac{f'(\kappa)[1 - g(l_s(\kappa))] (1-\alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[-1 - \frac{\phi(l_s(\kappa))}{1-l_s(\kappa)} \frac{1}{\phi'(l_s(\kappa))} \alpha(\kappa) \left[1 - \sigma^f(\kappa) \right] \right] \quad (10.3)$$

Note that:

$$\frac{\phi(l_s)}{1-l_s} \frac{1}{\phi'(l_s)} = \frac{g'(l_s)}{[1-g(l_s)]} \frac{1}{\frac{g'(l_s)(1-l_s)}{[1-g(l_s)]} \left[\frac{g"(l_s)}{g'(l_s)} - \frac{1}{1-l_s} + \frac{g'(l_s)}{[1-g(l_s)]} \right]} = \frac{1}{\left[\frac{g"(l_s)(1-l_s)}{g'(l_s)} - 1 + \phi(l_s) \right]} \quad (10.4)$$

Substituting the above equation in (10.3) yields:

$$\frac{\partial \left[f'(\kappa)[1 - g(l_s(\kappa))] - \delta - \rho \right]}{\partial \kappa} =$$

$$= \frac{f'(\kappa)[1 - g(l_s(\kappa))] (1-\alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[-1 - \frac{\alpha(\kappa) \left[1 - \sigma^f(\kappa) \right]}{\left[\frac{g"(l_s(\kappa))(1-l_s(\kappa))}{g'(l_s(\kappa))} - 1 + \phi(l_s(\kappa)) \right]} \right] =$$

$$= \frac{f'(\kappa)[1 - g(l_s(\kappa))] (1-\alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[-1 + \frac{\alpha(\kappa) \left[1 - \sigma^f(\kappa) \right]}{-\frac{g"(l_s(\kappa))(1-l_s(\kappa))}{g'(l_s(\kappa))} + \alpha(\kappa)} \right] =$$

$$= \frac{f'(\kappa)[1 - g(l_s(\kappa))] (1-\alpha(\kappa))}{\sigma^f(\kappa)\kappa} \left[\frac{\frac{g"(l_s(\kappa))(1-l_s(\kappa))}{g'(l_s(\kappa))} - \alpha(\kappa)\sigma^f(\kappa)}{-\frac{g"(l_s(\kappa))(1-l_s(\kappa))}{g'(l_s(\kappa))} + \alpha(\kappa)} \right] < 0$$

where in the third equality we use the equilibrium condition (5.2) and in the last inequality we use the assumption that states that $g(.)$ is strictly concave (so $-\frac{g"(l_s(\kappa))(1-l_s(\kappa))}{g'(l_s(\kappa))} > 0$). ■

10.4. Relationship between per capita income and productivity

10.4.1. Standard case:

When $l(\kappa^{ss}) = 1$ (the standard case) the effect of an increase in A over the steady state capital may be obtained by using the Implicit function Theorem over the Euler Equation at the steady state:

$$f'(\kappa^{ss}) - \frac{\delta+\rho}{A} \;=\; 0 \tag{10.5}$$

$$\frac{\partial\kappa^{ss}}{\partial A}\frac{A}{\kappa^{ss}} \;=\; \frac{\sigma^f(\kappa)}{(1-\alpha(\kappa))} \tag{10.6}$$

where we used equation (5.1). The effect of a change in productivity over the per capita income at the steady state $y^{ss} = Af(\kappa^{ss})$ is as follows:

$$\frac{\partial y^{ss}}{\partial A}\frac{A}{y^{ss}} \;=\; 1 + \left[\frac{f'(\kappa^{ss})\kappa^{ss}}{f(\kappa^{ss})}\right]\frac{\partial\kappa^{ss}}{\partial A}\frac{A}{\kappa^{ss}} \;=\; 1 + \frac{\alpha(\kappa)\sigma^f(\kappa)}{(1-\alpha(\kappa))}$$

10.4.2. Predation:

At the steady state the following condition should be satisfied:

$$f'(\kappa^{ss})[1 - g(l_s(\kappa^{ss}))] - \frac{\delta+\rho}{A} = 0 \tag{10.7}$$

Using the Implicit Function Theorem:

$$\frac{\partial\kappa^{ss}}{\partial A}\frac{A}{\kappa^{ss}} \;=\; \frac{1}{\left[\frac{(1-\alpha(\kappa^{ss}))}{\sigma^f(\kappa^{ss})} - \frac{g'(l_s(\kappa^{ss})l(\kappa^{ss})}{1-g(l_s(\kappa^{ss}))}\frac{\alpha(\kappa^{ss})}{\left[-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))}+\alpha(\kappa^{ss})\right]}\left[\frac{1-\sigma^f(\kappa^{ss})}{\sigma^f(\kappa^{ss})}\right]\right]} =$$

$$= \frac{\sigma^f(\kappa^{ss})}{(1-\alpha(\kappa^{ss}))}\frac{1}{\left[1 - \frac{\alpha(\kappa^{ss})}{\left[-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))}+\alpha(\kappa^{ss})\right]}[1-\sigma^f(\kappa^{ss})]\right]}$$

$$= \frac{\sigma^f(\kappa^{ss})}{(1-\alpha(\kappa^{ss}))}\left[1 + \frac{\alpha(\kappa^{ss})\left[1-\sigma^f(\kappa^{ss})\right]}{\left[-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))} + \alpha(\kappa^{ss})\sigma^f(\kappa^{ss})\right]}\right] > 0$$

where we use equations eqs. (5.1), (5.2), (10.2) and (10.4) in the first equality. It follows from (10.2) and the definition of $\phi(.)$ and equation (10.4) that:

$$\begin{aligned}
\frac{\partial l}{\partial \kappa}\frac{\kappa}{l} &= -\frac{\partial l_s}{\partial \kappa}\frac{\kappa}{l} = \\
&= \frac{l(\kappa)}{\frac{g'(l_s(\kappa))(1-l_s(\kappa))}{[1-g(l_s(\kappa))]}\frac{1}{(1-l_s(\kappa))}\left[-\frac{g"(l_s(\kappa))(1-l_s(\kappa))}{g'(l_s(\kappa))}+1-\phi(l_s)\right]}\frac{f'(\kappa)\kappa}{f(\kappa)}(1-\alpha(\kappa))\left[\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)}\right] = \\
&= \frac{\alpha(\kappa)}{-\frac{g"(l_s(\kappa))l(\kappa)}{g'(l_s(\kappa))}+\alpha(\kappa)}\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)} > 0 \qquad (10.8)
\end{aligned}$$

where in the last equality we used equation (5.2) and the definition of $\alpha(\kappa)$ and the equation $l + l_s = 1$. We turn now to analyze the effect of change in productivity over the per capita income $y^{ss} = Af(\kappa^{ss})l(\kappa^{ss})$:

$$\begin{aligned}
\frac{\partial y^{ss}}{\partial A}\frac{A}{y^{ss}} &= 1+\left[\frac{f'(\kappa^{ss})\kappa^{ss}}{f(\kappa^{ss})}+\frac{l'(\kappa^{ss})\kappa^{ss}}{l(\kappa^{ss})}\right]\frac{\partial \kappa^{ss}}{\partial A}\frac{A}{\kappa^{ss}} = \\
&= 1+\alpha(\kappa^{ss})\frac{\sigma^f(\kappa^{ss})}{(1-\alpha(\kappa^{ss}))}\left[\frac{1+\frac{1}{\left[-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))}+\alpha(\kappa^{ss})\right]}\left[\frac{1-\sigma^f(\kappa^{ss})}{\sigma^f(\kappa^{ss})}\right]}{1-\frac{\alpha(\kappa^{ss})\left[1-\sigma^f(\kappa^{ss})\right]}{\left[-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))}+\alpha(\kappa^{ss})\right]}}\right] = \\
&= 1+\frac{\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}{1-\alpha(\kappa^{ss})}+\frac{\alpha(\kappa^{ss})\left(1-\sigma^f(\kappa^{ss})\right)}{1-\alpha(\kappa^{ss})}\left[\frac{1+\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}{-\frac{g"(l_s(\kappa^{ss}))l(\kappa^{ss})}{g'(l_s(\kappa^{ss}))}+\alpha(\kappa^{ss})\sigma^f(\kappa^{ss})}\right] > 0
\end{aligned}$$

10.5. Proof of Proposition 8.1

Using equation (5.2):

$$\phi(l_s,\Gamma) = \frac{g'_{l_s}(l_s,\Gamma)(1-l_s)}{[1-g(l_s,\Gamma)]} = 1-\alpha \qquad (10.9)$$

$$\phi'_\Gamma(l_s,\Gamma) = \phi(l_s,\Gamma)\left[\frac{g^{"}_{l_s,\Gamma}(l_s,\Gamma)}{g'_{l_s}(l_s,\Gamma)}+\frac{g'_\Gamma(l_s,\Gamma)}{1-g(l_s,\Gamma)}\right] < 0 \qquad (10.10)$$

Using Implicit Function Theorem and equation (10.1):

$$\frac{\partial l_s}{\partial \Gamma} = -\frac{\phi'_\Gamma(l_s,\Gamma)}{\phi'_{l^s}(l_s,\Gamma)} < 0$$

since $\phi'_{l^s}(l_s, \Gamma) < 0$ by Lemma 1. ■

10.6. Effect of Γ on per capita income

At the steady state the following condition should be satisfied:

$$f'(\kappa^{ss})[1 - g(l_s(\kappa^{ss}, \Gamma), \Gamma)] - \delta - \rho = 0 \tag{10.11}$$

Using the Implicit Function Theorem:

$$\begin{aligned}
\frac{\partial \kappa^{ss}}{\partial \Gamma} \frac{\Gamma}{\kappa^{ss}} &= \frac{-\frac{g'_{\Gamma}(l_s(\kappa^{ss},\Gamma),\Gamma)\Gamma}{1-g(l_s(\kappa^{ss},\Gamma),\Gamma)} - \frac{g'_{l_s}(l_s(\kappa^{ss},\Gamma),\Gamma)}{1-g(l_s(\kappa^{ss},\Gamma),\Gamma)} \frac{\partial l_s(\kappa^{ss},\Gamma)}{\partial \Gamma}\Gamma}{\frac{f''(\kappa^{ss})\kappa^{ss}}{f'(\kappa^{ss})} - \frac{g'_{l_s}(l_s(\kappa^{ss},\Gamma),\Gamma)}{[1-g(l_s(\kappa^{ss},\Gamma),\Gamma)]} \frac{1}{\phi'(l_s)} \frac{f'(\kappa)\kappa^{ss}}{f(\kappa)} (1-\alpha(\kappa)) \left[\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)}\right]} = \\
&= \frac{-\frac{g'_{\Gamma}(l_s(\kappa^{ss},\Gamma),\Gamma)\Gamma}{1-g(l_s(\kappa^{ss},\Gamma),\Gamma)} + \frac{g'_{l_s}(l_s(\kappa^{ss},\Gamma),\Gamma) l(\kappa^{ss},\Gamma)}{1-g(l_s(\kappa^{ss},\Gamma),\Gamma)} \frac{\partial l(\kappa^{ss},\Gamma)}{\partial \Gamma} \frac{\Gamma}{l(\kappa^{ss},\Gamma)}}{\left[\frac{(1-\alpha(\kappa))}{\sigma^f(\kappa)} - \frac{g'_{l_s}(l_s(\kappa^{ss},\Gamma),\Gamma) l(\kappa)}{1-g(l_s(\kappa^{ss},\Gamma),\Gamma)} \frac{\alpha(\kappa)}{\left[-\frac{g''(l_s(\kappa))l(\kappa)}{g'(l_s(\kappa))} + \alpha(\kappa)\right]} \left[\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)}\right]\right]} = \\
&= \frac{\sigma^f(\kappa)}{(1-\alpha(\kappa))} \left[\varepsilon_{\Gamma}^{1-g}(\kappa^{ss}, \Gamma) + (1-\alpha(\kappa))\, \varepsilon_{\Gamma}^{l}(\kappa^{ss}, \Gamma)\right] \frac{1}{\left[1 - \frac{\alpha(\kappa)\left[1-\sigma^f(\kappa)\right]}{\left[-\frac{g''(l_s(\kappa))l(\kappa)}{g'(l_s(\kappa))} + \alpha(\kappa)\right]}\right]} = \\
&= \frac{\sigma^f(\kappa)}{(1-\alpha(\kappa))} \left[\varepsilon_{\Gamma}^{1-g}(\kappa^{ss}, \Gamma) + (1-\alpha(\kappa))\, \varepsilon_{\Gamma}^{l}(\kappa^{ss}, \Gamma)\right] \left[1 + \frac{\alpha(\kappa)\left[1-\sigma^f(\kappa)\right]}{\left[-\frac{g''(l_s(\kappa))l(\kappa)}{g'(l_s(\kappa))} + \alpha(\kappa)\sigma^f(\kappa)\right]}\right] > 0
\end{aligned}$$

where $\varepsilon_{\Gamma}^{1-g}(\kappa, \Gamma) = -\frac{g'_{\Gamma}(l_s(\kappa,\Gamma),\Gamma)\Gamma}{1-g(l_s(\kappa,\Gamma),\Gamma)} > 0$ is the elasticity of the fraction of income that goes to production factors with respect to Γ, $\varepsilon_{\Gamma}^{l}(\kappa, \Gamma) = \frac{\partial l(\kappa,\Gamma)}{\partial \Gamma} \frac{\Gamma}{l(\kappa,\Gamma)} > 0$ is the elasticity of labor with respect to Γ, which is positive since $\frac{\partial l(\kappa,\Gamma)}{\partial \Gamma} = -\frac{\partial l_s(\kappa,\Gamma)}{\partial \Gamma} > 0$ (see proposition 8.1). Note that in the first equality of equation we use eqs. (5.2), (10.2) and the equation $l + l_s = 1$; in the second equality we use eqs. (5.2) and (10.4) and; in the third equality we used equation (5.2).

We now turn to analyze the effect of the change in the institutional quality over the per capita income $y^{ss} = f(\kappa^{ss}) l(\kappa^{ss}, \Gamma)$:

$$\frac{\partial y^{ss}}{\partial \Gamma}\frac{\Gamma}{y^{ss}} = \left[\frac{f'(\kappa^{ss})\kappa^{ss}}{f(\kappa^{ss})} + \frac{\partial l(\kappa^{ss},\Gamma)}{\partial \kappa}\frac{\kappa^{ss}}{l(\kappa^{ss},\Gamma)}\right]\frac{\partial \kappa^{ss}}{\partial \Gamma}\frac{\Gamma}{\kappa^{ss}} + \frac{\partial l(\kappa^{ss},\Gamma)}{\partial \Gamma}\frac{\Gamma}{l(\kappa^{ss},\Gamma)} =$$

$$\left[\alpha(\kappa^{ss}) + \frac{\alpha(\kappa^{ss})}{-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})}\frac{1-\sigma^f(\kappa)}{\sigma^f(\kappa)}\right]\frac{\sigma^f(\kappa)\left[-\varepsilon_\Gamma^{1-g}(\kappa^{ss},\Gamma)+(1-\alpha(\kappa^{ss}))\varepsilon_\Gamma^l(\kappa^{ss},\Gamma)\right]}{(1-\alpha(\kappa^{ss}))}$$

$$\left[1 + \frac{\alpha(\kappa^{ss})\left[1-\sigma^f(\kappa)\right]}{\left[-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})\sigma^f(\kappa)\right]}\right] + \varepsilon_\Gamma^l(\kappa^{ss},\Gamma) =$$

$$\alpha(\kappa^{ss})\frac{\sigma^f(\kappa)\left[-\varepsilon_\Gamma^{1-g}(\kappa^{ss},\Gamma)+(1-\alpha(\kappa^{ss}))\varepsilon_\Gamma^l(\kappa^{ss},\Gamma)\right]}{(1-\alpha(\kappa^{ss}))} + \varepsilon_\Gamma^l(\kappa^{ss},\Gamma)+$$

$$\frac{\sigma^f(\kappa)\left[-\varepsilon_\Gamma^{1-g}(\kappa^{ss},\Gamma)+(1-\alpha(\kappa^{ss}))\varepsilon_\Gamma^l(\kappa^{ss},\Gamma)\right]\alpha(\kappa^{ss})\left[1-\sigma^f(\kappa)\right]}{(1-\alpha(\kappa^{ss}))\sigma^f(\kappa)}\left[\frac{\alpha(\kappa^{ss})\sigma^f(\kappa)}{\left(-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})\sigma^f(\kappa)\right)} + \right.$$

$$\left. + \left(\frac{1}{-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})}\right)\left(1 + \frac{\alpha(\kappa^{ss})\left[1-\sigma^f(\kappa)\right]}{\left[-\frac{g"(l_s(\kappa^{ss},\Gamma),\Gamma)l(\kappa^{ss},\Gamma)}{g'(l_s(\kappa^{ss},\Gamma),\Gamma)}+\alpha(\kappa^{ss})\sigma^f(\kappa)\right]}\right)\right] > 0$$

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